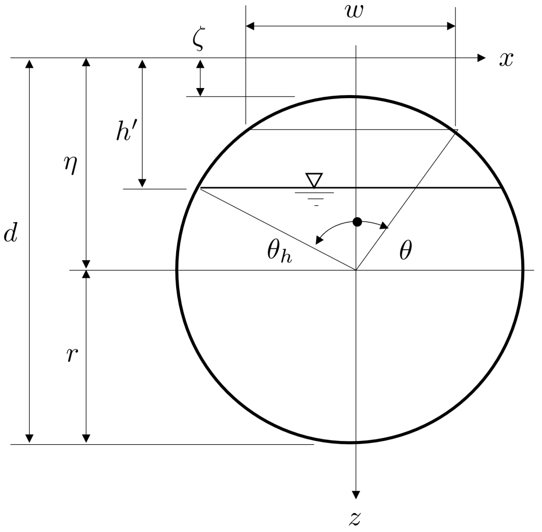

$$
\begin{aligned}
f(a) &= C\int_\zeta^a w(z)\sqrt{2\mathrm{g}z}\,\mathrm{d}z \\
&= C\sqrt{2\mathrm{g}}\int_0^{\theta_a}(2r\sin\theta)\sqrt{\zeta+r-r\cos\theta}\,(r\sin\theta\,\mathrm{d}\theta) \qquad \because\ \cos\theta_a = 1+\frac{\zeta-a}{r} \\
&= 2\,Cr^2\sqrt{2\mathrm{g}\eta}\int_0^{\theta_a}\sin^2\theta\,\left(1-\frac{r}{\eta}\cos\theta\right)^{\frac{1}{2}}\mathrm{d}\theta \qquad \because\ \eta=\zeta+r \\
&= 2\,Cr^2\sqrt{2\mathrm{g}\eta}\int_0^{\theta_a}\sin^2\theta\,\left[1-\frac{1}{2}\frac{r}{\eta}\cos\theta+\frac{1}{2}\sum_{k=2}^{\infty}\left\{\frac{1}{k!}\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-k+1\right)\left(-\frac{r}{\eta}\cos\theta\right)^k\right\}\right]\mathrm{d}\theta \\
&= 2\,Cr^2\sqrt{2\mathrm{g}\eta}\int_0^{\theta_a}\sin^2\theta\,\left(1-\frac{1}{2}\frac{r}{\eta}\cos\theta-\frac{1}{8}\frac{r^2}{\eta^2}\cos^2\theta-\frac{1}{16}\frac{r^3}{\eta^3}\cos^3\theta-\frac{5}{128}\frac{r^4}{\eta^4}\cos^4\theta-\cdots\right)\mathrm{d}\theta \\
&= Cr^2\sqrt{2\mathrm{g}\eta}\left[2I_0(\theta_a)-\frac{r}{\eta}I_1(\theta_a)-\frac{1}{4}\frac{r^2}{\eta^2}I_2(\theta_a)-\frac{1}{8}\frac{r^3}{\eta^3}I_3(\theta_a)-\frac{5}{64}\frac{r^4}{\eta^4}I_4(\theta_a)-\frac{7}{128}\frac{r^5}{\eta^5}I_5(\theta_a)-\frac{21}{512}\frac{r^6}{\eta^6}I_6(\theta_a)-\cdots\right] \\
&= Cr^2\sqrt{2\mathrm{g}\eta}\,\mathcal{A}(\theta_a)
\end{aligned}
$$

$$
\begin{aligned}
g(a) &= C\int_a^d w(z)\sqrt{2\mathrm{g}a}\,\mathrm{d}z = 2\,Cr^2\sqrt{2\mathrm{g}a}\int_{\theta_a}^{\pi}\sin^2\theta\,\mathrm{d}\theta = Cr^2\sqrt{2\mathrm{g}a}\,\Big[\theta-\sin\theta\cos\theta\Big]_{\theta_a}^{\pi} \\
&= Cr^2\sqrt{2\mathrm{g}a}\,\big(\pi-\theta_a+\sin\theta_a\cos\theta_a\big)
\end{aligned}
$$

$$
\begin{aligned}
Q &= f(d)-f(h')-g(h') = Cr^2\sqrt{2\mathrm{g}\eta}\Big[\mathcal{A}(\pi)-\mathcal{A}(\theta_h)\Big]-g(h') \\
&= Cr^2\sqrt{2\mathrm{g}\eta}\left[\pi\left(1-\frac{1}{32}\frac{r^2}{\eta^2}-\frac{5}{1024}\frac{r^4}{\eta^4}-\frac{105}{65536}\frac{r^6}{\eta^6}-\cdots\right)-\mathcal{A}(\theta_h)\right]-g(h')
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}Q}{\mathrm{d}\eta} &= \frac{1}{2}Cr^2\sqrt{\frac{2\mathrm{g}}{\eta}}\,\Big[\mathcal{A}(\pi)-\mathcal{A}(\theta_h)\Big]-\frac{1}{2}Cr^2\sqrt{\frac{2\mathrm{g}}{h'}}\big(\pi-\theta_h+\sin\theta_h\cos\theta_h\big) \qquad \because\ \Delta I_i = I_i(\pi)-I_i(\theta_h) \\
&\qquad + Cr^2\frac{\sqrt{2\mathrm{g}\eta}}{\eta}\left[\frac{r}{\eta}\Delta I_1+\frac{1}{2}\frac{r^2}{\eta^2}\Delta I_2+\frac{3}{8}\frac{r^3}{\eta^3}\Delta I_3+\frac{5}{16}\frac{r^4}{\eta^4}\Delta I_4+\frac{35}{128}\frac{r^5}{\eta^5}\Delta I_5+\frac{63}{256}\frac{r^6}{\eta^6}\Delta I_6+\cdots\right] \\
&= \frac{1}{2}\left[\frac{f(d)-f(h')}{\eta}-\frac{g(h')}{h'}\right]+Cr^2\sqrt{\frac{2\mathrm{g}}{\eta}}\,\left[-\frac{r}{\eta}I_1(\theta_h)+\frac{1}{2}\frac{r^2}{\eta^2}\left\{\frac{\pi}{8}-I_2(\theta_h)\right\}-\frac{3}{8}\frac{r^3}{\eta^3}I_3(\theta_h)\right. \\
&\qquad \left.+\frac{5}{16}\frac{r^4}{\eta^4}\left\{\frac{\pi}{16}-I_4(\theta_h)\right\}-\frac{35}{128}\frac{r^5}{\eta^5}I_5(\theta_h)+\frac{63}{256}\frac{r^6}{\eta^6}\left\{\frac{5\pi}{128}-I_6(\theta_h)\right\}+\cdots\right]
\end{aligned}
$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos^{n+1}\theta\sin\theta) = -(n+1)\cos^n\theta\sin^2\theta + \cos^{n+2}\theta = \cos^n\theta\left\{\cos^2\theta - (n+1)\sin^2\theta\right\}$$
$$= \cos^n\theta\left\{1-(n+2)\sin^2\theta\right\} = \cos^n\theta - (n+2)\cos^n\theta\sin^2\theta$$

$$\therefore \int_0^{\theta_a}\cos^n\theta\sin^2\theta\,\mathrm{d}\theta = \frac{1}{n+2}\left(\int_0^{\theta_a}\cos^n\theta\,\mathrm{d}\theta - \Big[\cos^{n+1}\theta\sin\theta\Big]_0^{\theta_a}\right)$$

$$\int\cos n\theta\,\mathrm{d}\theta = \frac{\sin n\theta}{n} = \begin{cases}\int\left(2\cos^2\theta-1\right)\mathrm{d}\theta = 2\int\cos^2\theta\,\mathrm{d}\theta - \theta & n=2\\ \int\left(4\cos^3\theta-3\cos\theta\right)\mathrm{d}\theta = 4\int\cos^3\theta\,\mathrm{d}\theta - 3\sin\theta & n=3\\ \int\left(8\cos^4\theta-8\cos^2\theta+1\right)\mathrm{d}\theta = 8\int\cos^4\theta\,\mathrm{d}\theta - 8\left(\frac{\theta}{2}+\frac{\sin2\theta}{4}\right)+\theta & n=4\\ \int\left(16\cos^5\theta-20\cos^3\theta+5\cos\theta\right)\mathrm{d}\theta & \\ \quad = 16\int\cos^5\theta\,\mathrm{d}\theta - 20\left(\frac{3}{4}\sin\theta+\frac{\sin3\theta}{12}\right)+5\sin\theta & n=5\\ \int\left(32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1\right)\mathrm{d}\theta = & \\ \quad = 32\int\cos^6\theta\,\mathrm{d}\theta - 48\left(\frac{3}{8}\theta+\frac{\sin2\theta}{4}+\frac{\sin4\theta}{32}\right)+18\left(\frac{\theta}{2}+\frac{\sin2\theta}{4}\right)-\theta & n=6\end{cases}$$

$$I_0(\theta_a) = \int_0^{\theta_a}\sin^2\theta\,\mathrm{d}\theta = \frac{1}{2}\Big(\theta_a - \cos\theta_a\sin\theta_a\Big)$$
$$I_1(\theta_a) = \int_0^{\theta_a}\cos\theta\sin^2\theta\,\mathrm{d}\theta = \frac{1}{3}\left(\int_0^{\theta_a}\cos\theta\,\mathrm{d}\theta - \Big[\cos^2\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{3}\Big(\sin\theta_a - \cos^2\theta_a\sin\theta_a\Big)$$
$$I_2(\theta_a) = \int_0^{\theta_a}\cos^2\theta\sin^2\theta\,\mathrm{d}\theta = \frac{1}{4}\left(\int_0^{a}\cos^2\theta\,\mathrm{d}\theta - \Big[\cos^3\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{4}\left(\frac{\theta_a}{2}+\frac{\sin2\theta_a}{4} - \cos^3\theta_a\sin\theta_a\right)$$
$$I_3(\theta_a) = \int_0^{\theta_a}\cos^3\theta\sin^2\theta\,\mathrm{d}\theta = \frac{1}{5}\left(\int_0^{a}\cos^3\theta\,\mathrm{d}\theta - \Big[\cos^4\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{5}\left(\frac{3}{4}\sin\theta_a+\frac{\sin3\theta_a}{12} - \cos^4\theta_a\sin\theta_a\right)$$
$$I_4(\theta_a) = \int_0^{\theta_a}\cos^4\theta\sin^2\theta\,\mathrm{d}\theta = \frac{1}{6}\left(\int_0^{a}\cos^4\theta\,\mathrm{d}\theta - \Big[\cos^5\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{6}\left(\frac{3}{8}\theta_a+\frac{\sin2\theta_a}{4}+\frac{\sin4\theta_a}{32} - \cos^5\theta_a\sin\theta_a\right)$$
$$I_5(\theta_a) = \int_0^{\theta_a}\cos^5\theta\sin^3\theta\,\mathrm{d}\theta = \frac{1}{7}\left(\int_0^{a}\cos^5\theta\,\mathrm{d}\theta - \Big[\cos^6\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{7}\left(\frac{10}{16}\sin\theta_a+\frac{5}{48}\sin3\theta_a+\frac{\sin5\theta_a}{80} - \cos^6\theta_a\sin\theta_a\right)$$
$$I_6(\theta_a) = \int_0^{\theta_a}\cos^6\theta\sin^4\theta\,\mathrm{d}\theta = \frac{1}{8}\left(\int_0^{a}\cos^6\theta\,\mathrm{d}\theta - \Big[\cos^7\theta\sin\theta\Big]_0^{\theta_a}\right) = \frac{1}{8}\left(\frac{10}{32}\theta_a+\frac{15}{64}\sin2\theta_a+\frac{3}{64}\sin4\theta_a+\frac{\sin6\theta_a}{192} - \cos^7\theta_a\sin\theta_a\right)$$